Quantum optics lecture
Part I: An Introduction to single photon physics

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Summary. In the first lecture, we discuss the quantum character of light. That there are light beams that cannot be split into weaker beams was first noticed by Planck, when he tried to explain the black body radiation. The concept of single photons was then introduced by Einstein in his seminal paper in 1905. Here, we won't look at the work of these two great physicists, who anyway get a lot of credit for their work in the current *year of physics*. Instead, we will have a look at some more recent linear optics experiments with single photons and more direct consequences and applications of the quantum character of light.

I. INTRODUCTION

This quantum optics lecture is part of the Photonics course at Imperial College London, which otherwise focusses on classical light. We start the course with a summary of the basic features of quantum mechanics and discuss them on the example of single photons. A photon is the smallest quantity in which light can occur. Photons behave like particles and their energy and momentum is given by

$$E = \hbar\omega \quad \text{and} \quad \mathbf{p} = \hbar\mathbf{k}\,. \tag{1}$$

Here ω is the frequency and $\mathbf{k}$ is the wave vector of the corresponding light field. In the following, we consider the effects of polarisation measurements, the generation of entanglement and look at recent linear optics experiments with single photons, which aim at testing the foundations of quantum mechanics and finding implementations for quantum information processing.

The main subject of quantum optics is the interaction between matter and light. A big part of this course will therefore be devoted to the generation of light from a laser driven single atom. Such a system can be described alternatively by rate equations, by the quantum jump approach or by the master equations. Afterwards we discuss more complex examples like atom-cavity systems and ion traps. For finding out more details about the course material, I recommend

- C. C. Gerry and P. L. Knight, *Introductory Quantum Optics*, Cambridge University Press (Cambridge, 2005).
- R. Loudon, *The Quantum Theory of Light*, Oxford University Press (Oxford, 2000).
- V. Vedral, *Lecture Course: Quantum Optics 2003/2004*. This course was given last year here at Imperial. The script can be down loaded (for free) from Vlatko's home page (http://vlatko.madetomeasure.biz/) at the University of Leeds.

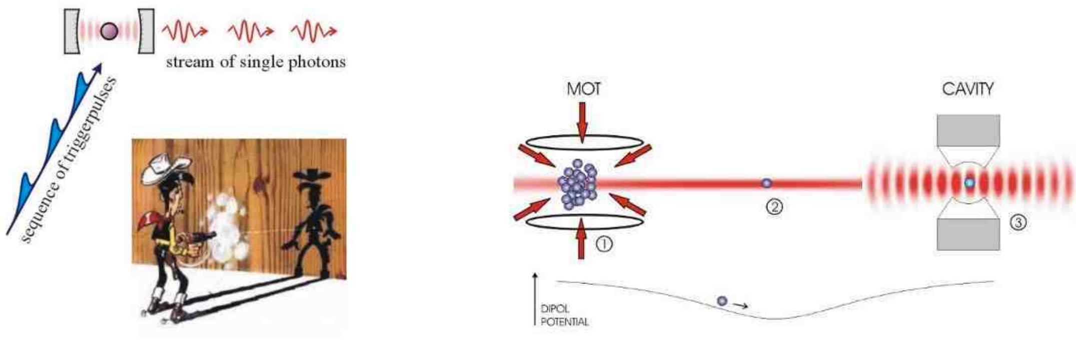

FIG. 1: Experimental setup for the generation of single photons on demand. The pictures are down loaded from the webpage of Rempe's group in Garching, who were the first to have an operating photon pistol. Alternatively, people experiment with photons from parametric down conversion and very weak laser fields.

For further reading, I recommend:

- M. O. Scully and M. S. Zubairy, *Quantum Optics*, Cambridge University Press (Cambridge, 1997).
- S. M. Barnett and P. M. Radmore, *Methods in Theoretical Quantum Optics*, Oxford University Press (Oxford, 1997).

Enjoy the course.

II. THE QUANTUM MECHANICS OF SINGLE PHOTONS

Single photons are difficult to generate and we see later during this lecture why. Only recently it has become feasible to generate single photons on demand, at least to a very good approximation. Such an experiment requires an atom-cavity setup, as shown in Figure 1. The atom guarantees the emission of at most one photon at the time, while the cavity contains an outcoupling mirror, which fixes the direction of the emitted light. For the moment, let us assume that we possess such an ideal source for creating single photons on demand.

A. Pure states

We see in the following, that a single photon cannot be described by a wave function $\psi(x,t)$ or $\tilde{\psi}(\omega,t)$ but has to be characterised by a quantum mechanical state vector $|\psi\rangle$ (if you are not familiar with this so-called Dirac notation, please look it up in an introductory quantum mechanics book). This state vector contains all the experimentally accessible information about the photon and can be used to predict the probability for all possible measurement outcomes. Examples of single photon observables are, for example, polarisation, arrival time at a detector, path information and frequency.

Each observable can be represented by a Hermitian operator $\mathbf{A}$ with real eigenvalues. The reason for this is, that the operator of an observable is constructed such that its eigenvectors $|n\rangle$ are the states, that yield the measurement outcome a_n with unit probability. The operator $\mathbf{A}$ can therefore be written as

$$\mathbf{A} = \sum_n a_n \, |n\rangle\langle n| \, . \tag{2}$$

For example, if $|\mathsf{H}\rangle$ and $|\mathsf{V}\rangle$ are one-photon states with horizontal or vertical polarisation, respectively, the observable for the polarisation $\mathbf{A}_{\rm pol}$ becomes

$$\mathbf{A}_{\rm pol} = |\mathsf{H}\rangle\langle\mathsf{H}| - |\mathsf{V}\rangle\langle\mathsf{V}| \, , \tag{3}$$

if the measurement outcome "+1" indicates horizontal and "−1" indicates vertical polarisation. Alternatively, one could have

$$\mathbf{A}_{\rm pol} = |\mathsf{R}\rangle\langle\mathsf{R}| - |\mathsf{L}\rangle\langle\mathsf{L}| \;\; \text{with} \;\; |\mathsf{R}\rangle = \big(|\mathsf{H}\rangle + \mathrm{i}\,|\mathsf{V}\rangle\big)/\sqrt{2} \;\; \text{and} \;\; |\mathsf{L}\rangle = \big(|\mathsf{H}\rangle - \mathrm{i}\,|\mathsf{V}\rangle\big)/\sqrt{2} \, , \tag{4}$$

where we consider a polarisation filter, whose setting "+1" indicates a different type of circular polarised light than the setting "−1."

Suppose a measurement device with the observable $\mathbf{A}$ given in Eq. (2) is entered by a single photon prepared in $|\psi\rangle$. Then the probability for finding outcome a_n equals

$$P_n = \big|\, \langle n|\psi\rangle \,\big|^2 = \langle\psi|n\rangle\langle n|\psi\rangle \, . \tag{5}$$

Calculating the expectation value of $\mathbf{A}$ given $|\psi\rangle$ is now straightforward and we find

$$\langle\mathbf{A}\rangle_\psi = \sum_n a_n \, P_n = \sum_n a_n \, \langle\psi|n\rangle\langle n|\psi\rangle = \langle\psi|\mathbf{A}\,|\psi\rangle \, . \tag{6}$$

Important to note is that in quantum mechanics, the state of a system after a measurement with outcome a_n equals, in general, $|n\rangle$. This is the only way to guarantee that repeating the same measurement on the same system for a second time yields the same result as the first measurement. Exceptions are, of course, possible and depend on the physical system and the concrete realisation of the measurement process.

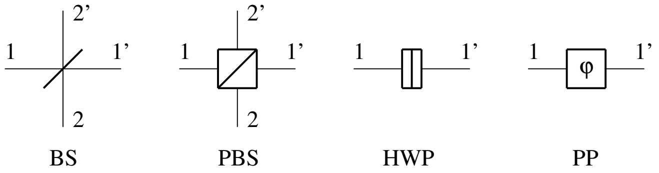

FIG. 2: Symbols for the presentation of linear optics elements in schematic experimental setups.

B. Mixed states

As long as the initial state and the history of a system are completely known, it can always be described by a pure state, i.e. by a state vector $|\psi\rangle$. However, in quantum optics we come across many examples, where it is only known that the system is with a certain probability prepared in a certain state but not which one. For instance, let's consider a laser driven two-level atom emitting photons at random times. If the emission times are not known, the atoms can only be described by a density matrix operator ρ. The main motivation for the introduction of this operator is, that it allows us to calculate the probability distribution and expectation values for measurements in such a situation without having to know the history of the system.

Suppose, a system is prepared with probability P_i in a state $|\psi_i\rangle$. Then its density matrix operator ρ equals

$$\rho = \sum_i P_i \, |\psi_i\rangle\langle\psi_i| \, . \tag{7}$$

If $|\psi_i\rangle$ is an N-dimensional state vector, then ρ is a positive and Hermitian $N \times N$ matrix with $\mathrm{Tr}(\rho) = 1$. Proceeding as above, we find in analogy to Eq. (5), that the probability for finding measurement outcome a_n equals

$$P_n = \sum_i P_i \, \langle\psi_i|n\rangle\langle n|\psi_i\rangle = \mathrm{Tr}\big(\, |n\rangle\langle n| \, \rho \, \big) \, . \tag{8}$$

Calculating the expectation value of $\mathbf{A}$ given ρ we now find

$$\langle \mathbf{A} \rangle_\rho = \sum_n a_n \, P_n = \mathrm{Tr}\big(\, \mathbf{A} \, \rho \, \big) \, , \tag{9}$$

which agrees with Eq. (6), in case of a pure state with $\rho \equiv |\psi\rangle\langle\psi|$. Note that the right hand side of Eqs. (8) and (9) can be calculated without knowing all the P_i and $|\psi_i\rangle$ but ρ.

III. LINEAR OPTICS EXPERIMENTS WITH SINGLE PHOTONS

To familiarise ourselves again with the quantum mechanical rules and to discuss the quantum nature of single photons, we now have a closer look at some recent linear optics experiments. Currently, many groups experiment with single photons (if you would like to know more, you can google them in the internet). Aim of these experiments are new fundamental tests of quantum mechanics and finding ways to implement quantum cryptography and quantum computing tasks.

Let's start by having a look at the basic elements of linear optics networks. Since it is almost impossible to create an interaction between photons, sufficiently strong non-linear optics elements for single photons are almost impossible to build. Linear optics elements are the 50 : 50 beam splitter, the polarising beam splitter, half way plates and phase plates. They redirect each photon *independently* to the output ports according to the following rules (for the notation of the input and output ports see Figure 2):

- 50 : 50 beam splitter (BS) or half transparent mirror: redirects each photon and adds a phase factor i with 50 % probability

$$\begin{aligned} \text{photon in input port 1 :} \quad & \alpha \, |\mathsf{H}\rangle_1 + \beta \, |\mathsf{V}\rangle_1 \;\longrightarrow\; \tfrac{1}{\sqrt{2}} \big(\alpha \, |\mathsf{H}\rangle_{1'} + \beta \, |\mathsf{V}\rangle_{1'} \big) + \tfrac{1}{\sqrt{2}} \, \mathrm{i} \big(\alpha \, |\mathsf{H}\rangle_{2'} + \beta \, |\mathsf{V}\rangle_{2'} \big) \\ \text{photon in input port 2 :} \quad & \alpha \, |\mathsf{H}\rangle_2 + \beta \, |\mathsf{V}\rangle_2 \;\longrightarrow\; \tfrac{1}{\sqrt{2}} \, \mathrm{i} \big(\alpha \, |\mathsf{H}\rangle_{1'} + \beta \, |\mathsf{V}\rangle_{1'} \big) + \tfrac{1}{\sqrt{2}} \big(\alpha \, |\mathsf{H}\rangle_{2'} + \beta \, |\mathsf{V}\rangle_{2'} \big) \end{aligned} \tag{10}$$

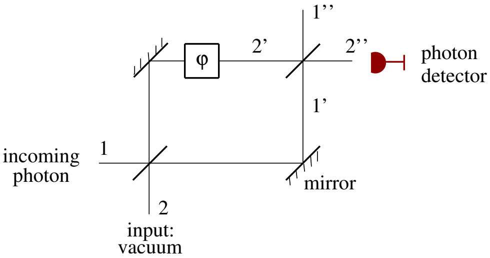

FIG. 3: Experimental setup for a one-photon interference experiment.

- polarising beam splitter (PBS): redirects photons in $|\mathsf{V}\rangle$ without affecting photons in $|\mathsf{H}\rangle$

$$\begin{aligned} \text{photon in input port 1 :} \quad & \alpha\,|\mathsf{H}\rangle_1 + \beta\,|\mathsf{V}\rangle_1 \;\longrightarrow\; \alpha\,|\mathsf{H}\rangle_{1'} + \beta\,|\mathsf{V}\rangle_{2'} \\ \text{photon in input port 2 :} \quad & \alpha\,|\mathsf{H}\rangle_2 + \beta\,|\mathsf{V}\rangle_2 \;\longrightarrow\; \alpha\,|\mathsf{H}\rangle_{2'} + \beta\,|\mathsf{V}\rangle_{1'} \end{aligned} \tag{11}$$

- half way plate (HWP): changes polarisation according to

$$\begin{aligned} \text{photon in input port 1 :} \quad & |+\rangle_1 \equiv \tfrac{1}{\sqrt{2}}\left(\alpha\,|\mathsf{H}\rangle_1 + \beta\,|\mathsf{V}\rangle_1\right) \;\longrightarrow\; |\mathsf{V}\rangle_{1'} \\ & |-\rangle_1 \equiv \tfrac{1}{\sqrt{2}}\left(\alpha\,|\mathsf{H}\rangle_1 - \beta\,|\mathsf{V}\rangle_1\right) \;\longrightarrow\; |\mathsf{H}\rangle_{1'} \end{aligned} \tag{12}$$

- phase plate (PP): adds an overall phase factor to the state of the incoming photon

$$\text{photon in input port 1 :} \quad \alpha\,|\mathsf{H}\rangle_1 + \beta\,|\mathsf{V}\rangle_1 \;\longrightarrow\; \mathrm{e}^{\mathrm{i}\varphi}\left(\alpha\,|\mathsf{H}\rangle_{1'} + \beta\,|\mathsf{V}\rangle_{1'}\right) \tag{13}$$

A. Interference of a single photon with itself

You are probably all very familiar with the interference of classical light. Also single photons can interfere. However, a single photon always interferes *only with itself* and the interpretation of interference effects is different from classical experiments although there are many similarities. As an example, let us have a look at the setup shown in Figure 3 and assume that a photon prepared in $|\psi\rangle$ enters input port 1, while input port 2 is entered by the vacuum state $|0\rangle$. Using Eqs. (10)-(13), we find that the initial state transforms according to

$$|\psi\rangle_1 \;\longrightarrow\; \tfrac{1}{\sqrt{2}}\,\mathrm{i}\left(|\psi\rangle_{1'} + \mathrm{i}\,\mathrm{e}^{\mathrm{i}\varphi}\,|\psi\rangle_{2'}\right) \tag{14}$$

before passing through the second beam splitter. (Here we assumed that a reflection on the mirror adds a phase factor i to the photon and omitted the vacuum states for the mode which do not contain any photons.) Taking the second beam splitter into account, the transformation (14) becomes

$$|\psi\rangle_1 \;\longrightarrow\; \tfrac{1}{2}\,\mathrm{i}\left(\left(1-\mathrm{e}^{\mathrm{i}\varphi}\right)|\psi\rangle_{1''} + \mathrm{i}\left(1+\mathrm{e}^{\mathrm{i}\varphi}\right)|\psi\rangle_{2''}\right). \tag{15}$$

We can now calculate the probability of finding a photon in output port $2''$ and obtain

$$P_{2''} \;=\; \left\|\,\tfrac{1}{2}\left(1+\mathrm{e}^{\mathrm{i}\varphi}\right)|\psi\rangle_{2''}\,\right\|^2 = \tfrac{1}{4}\left|1+\mathrm{e}^{\mathrm{i}\varphi}\right|^2 = \cos^2\left(\tfrac{1}{2}\varphi\right), \tag{16}$$

which oscillates between 0 and 1. As usual in quantum mechanical interference effects, there are different trajectories that contribute to the detection of the photon in the detector. Note that the photon travels simultaneously through the lower and the upper arm of the interferometer shown in Figure 3.

There are many more surprising quantum effects than a single photon interfering with itself. For example, one can show that two photons with the same wave vector $|\psi\rangle$, which enter a beam splitter simultaneously via the two different input ports always leave the setup through the same output port. This effect — the vanishing probability for two photons in different outputs — is known as the Hong-Ou-Mandel dip.

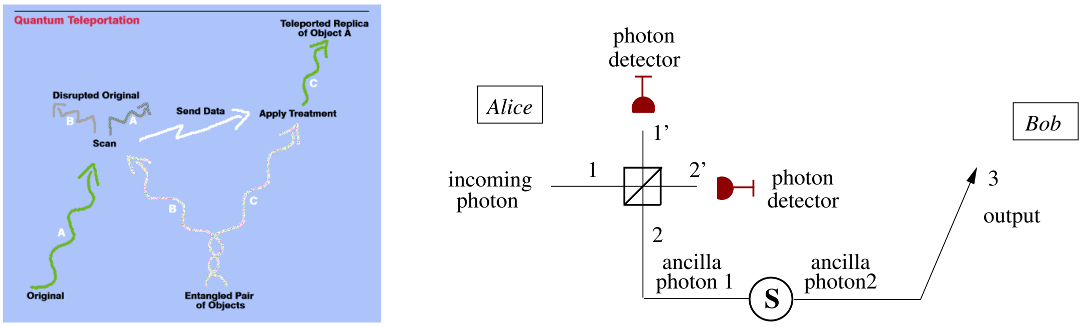

FIG. 4: Schematic view and experimental setup for a teleportation experiment of a single photon. Einstein thought, that this was not possible.

B. A maximally entangled two-photon state

An entangled state is a state whose vector $|\psi\rangle$ cannot be written as a product of the single states of its subsystems. An example is the two photon state that is obtained by sending two photons through a 50 : 50 beamsplitter, as shown in Figure 2. Suppose, one input port of the BS is entered by a photon prepared in $|\mathsf{H}\rangle$ and the other one is entered by a photon prepared in $|\mathsf{V}\rangle$. Then one can show using Eq. (10) that

$$\begin{aligned} |\mathsf{H}\rangle_1|\mathsf{V}\rangle_2 &\longrightarrow \tfrac{1}{2}\left(|\mathsf{H}\rangle_{1'}+\mathrm{i}|\mathsf{H}\rangle_{2'}\right)\left(|\mathsf{V}\rangle_{2'}+\mathrm{i}|\mathsf{V}\rangle_{1'}\right) \\ &= \tfrac{1}{2}\left(|\mathsf{H}\rangle_{1'}|\mathsf{V}\rangle_{2'}-|\mathsf{H}\rangle_{2'}|\mathsf{V}\rangle_{1'}+\mathrm{i}\left(|\mathsf{H}\rangle_{1'}|\mathsf{V}\rangle_{1'}+|\mathsf{H}\rangle_{2'}|\mathsf{V}\rangle_{2'}\right)\right). \end{aligned} \tag{17}$$

Under the condition of the detection of one photon per output port, which happens with probability $\frac{1}{2}$ the state is projected onto

$$|\mathsf{H}\rangle_1|\mathsf{V}\rangle_2 \longrightarrow \tfrac{1}{2}\left(|\mathsf{H}\rangle_{1'}|\mathsf{V}\rangle_{2'}-|\mathsf{H}\rangle_{2'}|\mathsf{V}\rangle_{1'}\right), \tag{18}$$

which is a maximally entangled state. This becomes obvious, if we look at the correlations of polarisation measurements performed in output ports $1'$ and $2'$. Detecting $|\mathsf{H}\rangle$ in one port is always correlated to the detection of a photon in $|\mathsf{V}\rangle$ in the other output port. Remarkably, this strong correlations apply also to measurements in the $|\pm\rangle$-basis introduced in Eq. (12). The generated correlations therefore have no classical analog.

C. Teleportation of photons

One of the most important applications of maximally entangled photon states is quantum cryptography. If you want to learn more about that, then google "BB84" in the internet. This is the name of the protocol that Bennett and Brassard proposed in 1984 in order to establish a shared secret key between Alice and Bob via the transmission of single photons. Here we have a look at the not less famous *teleportation of a single photon*, which has been proposed by six scientists, including Bennett, in 1993. The first photon teleportation experiment was performed by Zeilinger's group in Innsbruck and by DeMartini's group in Rom in 1997.

Teleportation is the name given by science fiction writers to the feat of making an object or person disintegrate in one place while a perfect replica appears somewhere else. Perfect teleportation of quantum states is indeed possible but not without destroying the original. In the past, the idea of teleportation was not taken very seriously, because it was thought to violate the uncertainty principle of quantum mechanics, which forbids any measuring or scanning process from extracting all the information about the quantum state. The more accurately an object is scanned, the more it is disturbed by the scanning process, until one reaches a point where the object's original state has been completely disrupted, still without having extracted enough information to make a perfect replica. This sounds like a solid argument against teleportation: if one cannot extract enough information from an object to make a perfect copy, it would seem that a perfect copy cannot be made.

The key feature that helps to get around this is entanglement. As Figure 4 suggests, the unscanned part of the information is conveyed from Alice to Bob via a maximally entangled ancilla photon pair. Suppose, Alice's photon is initially prepared in

$$\alpha\,|\mathsf{H}\rangle_1+\beta\,|\mathsf{V}\rangle_1\,, \tag{19}$$

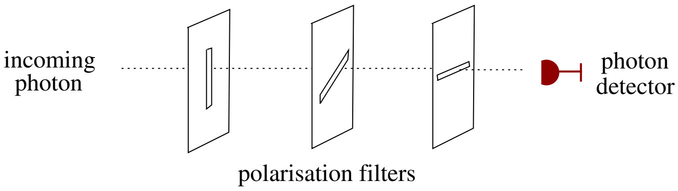

FIG. 5: Setup for a demonstration of the inverse quantum Zeno effect with linear optics involving three polarisation filters.

while the state of the ancillas equals

$$\frac{1}{\sqrt{2}} \left(|\mathsf{H}\rangle_2 |\mathsf{H}\rangle_3 + |\mathsf{V}\rangle_2 |\mathsf{V}\rangle_3 \right) . \tag{20}$$

The teleportation is successful, if Alice measures one photon in each output port. The state that contributes to this is

$$\frac{1}{\sqrt{2}} \left(\alpha \, |\mathsf{H}\rangle_1 |\mathsf{H}\rangle_2 |\mathsf{H}\rangle_3 + \beta \, |\mathsf{V}\rangle_1 |\mathsf{V}\rangle_2 |\mathsf{V}\rangle_3 \right) . \tag{21}$$

To decouple the photons in output ports $1'$ and $2'$ from Bob's photon in outport 3, Alice should perform measurements in the $|\pm\rangle$-basis introduced in Eq. (12). If she find the photons for example in $|+\rangle_{1'}|+\rangle_{2'}$, Bob obtains a photon in

$$\alpha \, |\mathsf{H}\rangle_3 + \beta \, |\mathsf{V}\rangle_3 \, , \tag{22}$$

which is indistinguishable from Alice's photon. In some cases, Bob has to perform a phase flip of the photon state. This can be done without knowing the state and Alice can tell him whenever this is necessary (for example in case she detects $|+\rangle_{1'}|-\rangle_{2'}$).

Of course, one could argue that Alice could simply send the photon directly to Bob, instead of teleporting it, but there might be a big distance between them. Teleportation is also an important ingredient for linear optics quantum computing.

D. The inverse quantum Zeno effect

Another quantum mechanical effect with no classical analog is the inverse quantum Zeno effect. It predicts that a system, which would normally not evolve in time, can be made to follow a certain trajectory by applying rapidly repeated measurements, which "continuously" test whether the system follows the trajectory or not. With photons, this effect can be tested easily and a possible setup for such an experiment is shown in Figure 5. Suppose, an incoming photon passes trough a filter for vertical polarisation. Then there is no chance that it also passes through the polarisation filter on the right, which measures horizontal polarisation. However, if there are $N > 0$ polarisation filters placed between the two filters, which measure, whether the photon rotates from $|\mathsf{V}\rangle$ to $|\mathsf{H}\rangle$, the probability to have a photon coming out at the end is no longer zero. For $N \to \infty$, this probability is one and the photon actually rotates on its way from the first to the last filter by 90^0.

Can you show this for the case where the N additional polarisation filters are placed at equal distances from each other and all vary in their orientation by an angle φ_N from the next neighbours? What φ_N would you choose?

IV. FINAL REMARKS

In this lecture, we discussed the quantum mechanical features, which dominate the behaviour of light on the level of single photons. Single photons have to be described by a state vector $|\psi\rangle$ and are characterised by properties like their frequency, polarisation, path and arrival time. We saw that a single photon can interfere with itself inside a linear optics network. Sending two photons through a beam splitter can result in the generation of a maximally entangled state, with applications in quantum teleportation. Frequent measurements can affect the state of a photon such that it follows a certain trajectory (inverse quantum Zeno effect).