QUPREST 2006

Focus meeting: Quantum Process Estimation

Budmerice, Slovakia
27th September - 1st October 2006

Qubit Applications

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QUPREST 2006

Experimental capabilities grow to encompass the precise control of ever larger quantum systems, issues surrounding the efficient execution of quantum process and state estimation become very significant. The ”testing” of quantum systems may pose to be a major hurdle in the proofing of ”engineered” quantum systems in the future. We feel that a focused meeting on this topic will allow the community to pin point potential future problems and flag potential solutions.

Our meeting is a part of a series of research workshops organized within a European Union integrated project called QAP (Quantum applications). The meeting will be focused on various aspects of quantum measurement theory and namely of problems of state and process reconstruction from incomplete experimental data.

Place:
Address: Domov slovenskych spisovatelov, Kaštiel, 90086 Budmerice, Slovakia,
Tel.: ++421 33 6448289

Conference venue:
Budmerice chateaux is located roughly 50 km northeast from Bratislava. It was built in 19th century by Palfy's family. Nowadays it is a property of Slovak writers Association. The chatreaux is surrounded by a big park.

Invited speakers:
⋆ Emili Bagan (Barcelona, Spain)
⋆ Janos Bergou (New York, USA)
⋆ Berge Englert (Singapore)
⋆ Richard Gill (Utrecht, Netherland)
⋆ Masahito Hayashi (Tokyo, Japan)
⋆ Lorenzo Maccone (Pavia, Italy)
⋆ Denes Petz (Budapest, Hungary)
⋆ Ferdinand Schmidt-Kaler (Ulm, Germany)
⋆ Christof Wunderlich (Siegen, Germany)

Organizers:
⋆ Vladimír Bužek (RCQI, FI MU)
⋆ Jason Twamley (RCQI, FI MU)
⋆ Mário Ziman (RCQI, FI MU)

Conference Program

WEDNESDAY, 27.9.2006

17:00	conference bus Bratislava - Budmerice
19:30	Registration and accommodation
20:00	Welcome dinner
21:00	Common evening

THURSDAY, 28.9.2006

08:30	Breakfast
	MORNING SESSION chaired by Mário Ziman
09:00-10:15	Christof Wunderlich: Quantum state and process estimation with trapped ions
10:15-10:45	Holger F.Hoffman: How to identify errors in quantum operations
10:45-11:15	Coffee break and refreshment
11:15-12:15	Emili Bagan: Local vs. Collective Measurements in Quantum State Estimation and Discrimination
12:15-12:45	Masoud Mohseni: Direct Characterization of Quantum Dynamics
13:00-13:30	Lunch
	AFTERNOON SESSION chaired by Janos Bergou
15:00-16:00	Richard Gill: Conciliation of Bayesian and Pointwise Optimal State Estimation
16:00-16:30	Joseph Emerson: Efficient randomization protocol for direct estimation of multi-body noise probabilities
16:30-17:00	Fernando Brandao: Quantitative Entanglement Witnesses
17:00-17:30	Tea break and refreshment
17:30-18:00	Denes Petz: State estimation for qubits
18:00-18:30	Matthias Kleinman: Can unambiguous discrimination of mixed states always be reduced to the case of pure states?
18:30-19:00	Giulio Chiribella: Quantum information becomes classical when distributed to many users
19:00-19:30	Dinner
20:30	Common evening

Conference Program

FRIDAY, 29.9.2006

08:30-09:00	Breakfast
	MORNING SESSION chaired by Chris Wunderlich
09:00-10:15	Janos Bergou: Discrimination of quantum states with selected applications
10:15-10:45	Alexei Gilchrist: Quantum Process Tomography and Metrics
10:45-11:15	Coffee break
11:15-12:15	Ferdinand Schmidt-Kaler: Entangled states and State tomography in an elementary ion-based quantum processor
12:15-12:45	Jonas Kahn: Fast estimation of $SU(d)$ operation
12:45-13:15	Lunch
13:30-18:30	 Conference trip to castle ”Červený kameň” (Red Stone) and visit of the falcon performance.
19:30-24:00	Conference banquet

Conference Program

SATURDAY, 30.9.2006

08:30-09:00	Breakfast
	MORNING SESSION chaired by Denes Petz
09:00-10:15	Lorenzo Maccone: Quantum tomography: complete characterization of states and processes
10:15-10:45	Matteo Paris: Photon statistics without photon counting and applications
10:45-11:15	Coffee break
11:15-12:15	Masahito Hayashi: Statistical analysis on testing of an entangled state based on Poisson distribution framework
12:15-12:45	Robin Blume-Kohout: Bayesian Mean Estimation: accurate and honest state estimation
13:00-13:30	Lunch
	AFTERNOON SESSION chaired by Vladimír Bužek
15:00-16:00	Berge Englert: Minimal qubit tomography, the Singapore Protocol for QKD, and optimal POVMs
16:00-16:30	Anthony Chefles: Unambiguous discrimination among oracle operators
16:30-17:00	Massimiliano Sacchi: Information-disturbance tradeoff in estimating maximally entangled states and in quantum state discrimination
17:00-17:30	Tea break
17:30-18:00	Matthijs Branderhorst: Convex optimization for quantum state and process tomography
18:00-18:30	Michal Karpinski: Depolarizing quantum channels
18:30-19:00	Stefano Olivares: Cloning of Gaussian states by linear optics
19:00-19:30	Dinner
20:30-22:00	Round table discussion (chaired by Vladimír Bužek)

SUNDAY, 1.10.2006

09:00-09:30	Breakfast
10:00	Conference bus to Bratislava (Airport [10:50], Bus station [11:10], Petržalka Railway Station [11:30])

List of participants

1. Emili Bagan (Universita Autonoma de Barcelona, Spain)
2. Janos Bergou (CUNY, New Your, USA)
3. Robin Blume-Kohout (Caltech - IQI , Pasadena, USA)
4. Fernando Brandao (Imperial College London, United Kingdom)
5. Matthijs Branderhorst (Oxford University, United Kingdom)
6. Vladimr Bužek (RCQI, Bratislava, Slovakia)
7. Anthony Chefles (Hewlett-Packard Laboratories, Bristol, UK)
8. Giulio Chiribella (University of Pavia, Italy)
9. Berge Englert (Department of Physics, National University of Singapore)
10. Richard Gill (Mathematical Institute, University Utrecht, Netherland)
11. Alexei Gilchrist (University of Queensland, Brisbane, Australia)
12. Joseph Emerson (University of Waterloo, Canada)
13. Masahito Hayashi (University of Tokio, Japan)
14. Holger F.Hofmann (JST-CREST, Hiroshima University)
15. Jonas Kahn (Universit Paris-Sud , Orsay CEDEX, France)
16. Michal Karpinski (Warsaw Univeristy, Poland)
17. Matthias Kleinmann (Heinrich-Heine-Universitt Dsseldorf, Germany)
18. Lorenzo Maccone (University of Pavia, Italy)
19. Derek McHugh (RCQI, Bratislava, Slovakia)
20. Masoud Mohseni (University of Toronto, Canada)
21. Alex Monras (FAE, Universitat Autonoma de Barcelona, Spain)
22. Stefano Olivares (Universita di Milano, Milano, Italy)
23. Matteo Paris (Universita di Milano, Milano, Italy)
24. Denes Pétz (Eotves University, Budapest, Hungary)
25. Martin Plesch (RCQI, Bratislava, Slovakia)
26. Peter Rapčan (RCQI, Bratislava, Slovakia)
27. Daniel Reitzner (RCQI, Bratislava, Slovakia)
28. Massimiliano Sacchi (University of Pavia, Italy)
29. Ferdinand Schmidt-Kaler (Universitet Ulm, Germany)
30. Michal Sedlák (RCQI, Bratislava, Slovakia)
31. Peter Štelmachovič (RCQI, Bratislava, Slovakia)
32. Christof Wunderlich (University of Siegen, Germany)
33. Mário Ziman (RCQI, Bratislava, Slovakia)

Abstracts

1. Emilio Bagan
   Local vs. Collective Measurements in Quantum State Estimation and Discrimination
   State estimation and discrimination when a number of uncorrelated copies of the states is available is a fundamental issue in quantum information. Quantum measurements, as those required by the estimation/discrimination protocols, fall into two categories: collective and local. The former provide the best guess for the unknown state, but are hard to implement experimentally.

   In this talk we raise the question of whether local measurements, though allegedly less powerful, can perform as well as collective ones. The answer depends very much on the particular scenario under consideration. For qubit mixed state estimation, we show that there is a gap between separable protocols (a useful generalization of the local ones) and collective protocols in the limit of a large number of copies. Moreover, we give a local protocol that saturates the separable bound.

   We show that for correlated copies (such as those produced by a cloning machine) the situation can be dramatically worse: the best separable measurements may not attain perfect determination even when an infinite number of copies is available. This is used to device a secret direction-sharing protocol.

   In multiple-copy two-state discrimination, the question above has a positive answer for pure states and for any number of copies. This is an open question for mixed states. We show, however, that fixed local measurements fall way behind collective ones. We comment on a quantum version of the Chernoff bound on the probability of error. We discuss some implications of our results concerning the distance and metric on the space of states induced by the probability of error.

2. Janos Bergou
   Discrimination of quantum states with selected applications
   There are various strategies for the discrimination of known quantum states that are optimal with respect to some reasonable criteria. An overview of the state of the art of state discrimination is given in our recent review paper [1]. State discrimination has applications to quantum key distribution, eavesdropping and, in general, in the read-out stage of quantum information processing and quantum computing. We discuss the usefulness of optimized strategies by analyzing the B92 quantum key distribution protocol [2].

   In all these works it has been assumed that the states to be discriminated must be known, i.e., they must be drawn from a set of known states, since the discrimination itself is a (generalized) measurement that is specific to the states. Recently we considered state discrimination problems in which all [3] or some of the states are unknown [4] and copies of the unknown states are provided as program (programmable state discrimination). There is a close connection between the discrimination of unknown pure states and the discrimination of known mixed sates and techniques that were developed for the discrimination of mixed states [5] are directly applicable to the problem of programmable state discrimination. Furthermore, we have suggested and realized linear optical implementations of the optimized generalized measurements (POVMs) experimentally [6].

The problem of operator testing, which includes the testing of quantum gates, is a frequent task in quantum information processing [7]. We demonstrate that it is closely related to the discrimination of quantum states. In order to discriminate between operators, particles are prepared in some reference state, called the fiducial state, at the input. These states are then transformed by the different operators, and measurements are performed on the resulting states at the output, so that one is ultimately discriminating between quantum states. We present generalized measurements and their linear optical implementations that accomplish this task optimally.

References:
1. J. Bergou, U. Herzog, and M. Hillery in Quantum State Estimation, edited by M. G. A. Paris and J. Rehacek, Lecture Notes in Physics 649, 297 (Springer, New York, 2004).
2. C. H. Bennett, Quantum cryptography using any two nonorthogonal states, Phys. Rev. Lett. 68, 3121 (1992).
3. J. Bergou and M. Hillery, Universal programmable quantum state discriminator that is optimal for unambiguously distinguishing between unknown states, Phys. Rev. Lett. 94, 160501 (2005).
4. J. Bergou, V. Buzek, E. Feldman, U. Herzog, and M. Hillery, Programmable state discriminators with simple programs, Phys. Rev. A 73, 062334 (2006).
5. U. Herzog and J. Bergou, Unambiguous discrimination of mixed quantum states, Physical Review A 71, 050301R (2005); J. Bergou, E. Feldman, and M. Hillery, Unambiguous discrimination of two subspaces, Physical Review A 73, 032107 (2006).
6. M. Mohseni, A. Steinberg, and J. Bergou, Optical realization of optimal unambiguous discrimination for pure and mixed quantum states, Phys. Rev. Lett. 93, 200403 (2004).
7. See G. M. D' Ariano and P. L. Presti in Quantum State Estimation, edited by M. G. A. Paris and J. Rehacek, Lecture Notes in Physics 649, 297 (Springer, New York, 2004).

3. Robin Blume-Kohout

Bayesian Mean Estimation: accurate and honest state estimation

I will present a new procedure for quantum state or process estimation, Bayesian mean estimation (BME). Unlike current procedures, BME is suitable for fault-tolerant quantum hardware verification. Fault-tolerant quantum computing requires that states and gates be predictable to within a fault-tolerance threshold of between 10^{-2} and 10^{-8}. Maximum-likelihood estimation (MLE), the prevailing state estimation procedure, typically yields estimates that have zero eigenvalues. These zero probabilities do not honestly represent the estimatoŕs knowledge, nor are they compatible with reliable error estimation. Bayesian mean estimation avoids this pitfall. Furthermore, the BME estimate optimizes every operational measure of accuracy.

4. Fernando Brandao

Quantitative Entanglement Witnesses

Entanglement witnesses provide tools to detect entanglement in experimental situations without the need of having full tomographic knowledge about the state. If one estimates in an experiment an expectation value smaller than zero, one can directly infer that the state has been entangled, or specifically multi-partite entangled, in the first place. In this talk, we emphasize that all these tests – based on the very same data - give rise to quantitative estimates in terms of entanglement measures: "if a test is strongly violated, one can also infer that the state was quantitatively very much entangled". We consider various measures of entanglement, including the negativity, the entanglement of formation, and the robustness of entanglement, in the bipartite and multipartite setting. As examples, we discuss several experiments in quantum state preparation that have recently been performed.

5. Matthijs Branderhorst
   Convex optimization for quantum state and process tomography
   We reconstruct the vibrational density matrix of diatomic potassium molecules using convex optimization. Because the optimal experiment design is also a convex optimization problem we estimate the distribution of experimental settings that maximizes the efficiency of the state estimation with an accuracy that can be chosen in advance. The same technique can be applied to quantum process tomography to estimate the decoherence process in potassium molecules which takes place due to coupling to rotational degrees of freedom.

6. Anthony Chefles
   Unambiguous discrimination among oracle operators
   In quantum computing, unitary oracle operators are conventionally used for computing functions. Thus, the problem of distinguishing among functions reduces to the problem of distinguishing among the corresponding oracle operators. Here, we provide new insights into this general problem, for two oracle models: standard and minimal (or erasing) oracles. General conditions for unambiguous discrimination among such oracles are obtained, and detailed investigations of oracles of particular importance, such as the Grover oracle which arises in fast quantum searching, are carried out. In addition, remarkable formal relationships between the standard and minimal oracle sets are obtained, which reveal them to be equivalent in various senses regarding their distinguishability properties.

7. Giulio Chiribella
   Quantum information becomes classical when distributed to many users.
   Any physical transformation that equally distributes quantum information to a certain number M of users can be approximated with a classical procedure, where the input system is measured, the measurement outcome is broadcast, and each user prepares a local state accordingly. In particular, the optimal cloning of pure and mixed states can be approximately realized via quantum state estimation. The accuracy the approximation is of order 1/M in the number of users, hence, for large M, there is no advantage of a coherent quantum information processing over a classical incoherent procedure.

8. Berge Englert
Minimal qubit tomography, the Singapore Protocol for QKD, and optimal POVMs
Minimal qubit tomography, which uses highly symmetric four-output POVMs, enables one to realize a qubit protocol for quantum key distribution presented that is fully tomographic and more efficient than other tomographic protocols. Under ideal circumstances the efficiency is $\log_2(4/3) = 0.415$ key bits per qubit sent, which is 25% more than the efficiency of $1/3 = 0.333$ for the standard 6-state protocol. One can extract 0.4 key bits per qubit by a simple two-way communication scheme, and can so get close to the information-theoretical limit. I'll discuss the noise thresholds for secure key bit generation in the presence of unbiased noise. The analysis of one important eavesdropping scenario requires the determination of the optimal POVM for accessing the information leaked to the eavesdropper. I will remark on this optimization problem whose solution is central to the analysis.

9. Richard Gill
Conciliation of Bayesian and Pointwise Optimal State Estimation
I'll derive a sharp asymptotic bound on mean fidelity when N identical quantum systems, whose state depends on a vector of unknown parameters, are jointly measured, and the state estimated, in any way whatever. The bound is an integrated version of Holevo's (1982) quantum Cramer-Rao bound. This connects fixed N exact Bayesian optimality pursued in physics, with pointwise asymptotic optimality favoured in statistics. In some important examples I will show asymptotic optimality of interesting and useful measurement-and-estimation schemes. On the way I obtain a new family of "dual Holevo bounds". I hope also to remark on the connections with highly related recent work of Hayashi and Matsumoto, and of Guta and Kahn.

10. Alexei Gilchrist
Quantum Process Tomography and Metrics
With our increasing ability to fashion devices to process quantum information the question of characterising those devices becomes more poignant. There is an exponential cost in system size to fully determine a physical process via process tomography. In addition to the large number of measurements that are necessary to acquire the data, performing maximum-likelihood tomography to compensate for noisy data is itself a computationally intensive task, further compounding the problem. We will present two complementary approaches to this problem: the first is to recast the maximum-likelihood tomography as a semi-definite program that allows relatively efficient numerical computation. The second is to identify key metrics that are useful and can be obtained directly without full process tomography.

11. Joseph Emerson
Efficient randomization protocol for direct estimation of multi-body noise probabilities
Quantum process estimation is essential for characterizing the quality of quan-

tum information processors and quantum communication channels. In particular the selection and optimization of error-correction strategy requires knowledge of certain key parameters of the noise. I will describe a family of efficient randomization protocols I have recently developed, based on twirling the quantum bits with the Clifford group, that allow direct estimation of the distinct probability weights for each k-body error affecting a quantum channel or quantum memory. These protocols provide an exponential resource savings relative to quantum process tomography for noise characterization. These protocols may be readily implemented with the degree of control currently available in NMR and atomic traps. Time-permitting I will present preliminary results from liquid and solid state NMR implementations of these noise-estimation protocols.

12. Masahito Hayashi, Akihisa Tomita, Keiji Matsumoto
Statistical analysis on testing of an entangled state based on Poisson distribution framework
A hypothesis testing scheme for entanglement has been formulated based on the Poisson distribution framework instead of the POVM framework. Three designs were proposed to test the entangled states in this framework. The designs were evaluated in terms of the asymptotic variance. It has been shown that the optimal time allocation between the coincidence and anti-coincidence measurement bases improves the conventional testing method. The test can be further improved by optimizing the time allocation between the anti-coincidence bases.

13. Holger F. Hofmann
How to identify errors in quantum operations: a peak into the "black box" of process tomography
To navigate the vastness of Hilbert space, it is useful to consider the actual observable effects expressed by the abstract formalism. The mathematical complexity can then be reduced considerably by a more direct identification of process components with their physical meaning. In this presentation, I will explain how the measurement of spin flips in the complementary X and Z bases can be used to analyze the essential features of a quantum process without the need to perform full quantum process tomography. To illustrate the method, the experimental results obtained from an optical quantum controlled-NOT gate realized at Hokkaido University (Phys. Rev. Lett. 95, 210506 (2005)) will be discussed. In particular, it can be shown that the measurement data of only two classical logic operations performed by the gate is sufficient to estimate the entanglement capability and the usefullness of the gate as a Bell analyzer. The results of the analysis demonstrate that a more intuitive approach to quantum process analysis is possible and may help to reduce the mathematical "overhead" of experimental tests of quantum processes. Hopefully, this kind of analysis will eventually lead us to a better understanding of the physics involved in quantum information.

14. Jonas Kahn
Fast estimation of $SU(d)$ operation
An unknown operation (channel) U can be evaluated by sending input states through the channel and measuring the output states. An interesting phenomenon

is that $1/N2$ rate can be achieved instead of the $1/N$ common in statistics, using entanglement between input states. This was already known for $U \in SU(2)$. This talk is on the proof for $U \in SU(d)$, for general d.

15. Michal Karpinski
Depolarizing quantum channels
We report on the experimental realization of a series of depolarizing quantum channels acting on the polarization state of a single photon. Depolarization was implemented via a random application of unitary transformations with appropriate probability distributions using an electrically-driven polarization controller. The quantum channels were characterized using Entanglement-Enhanced Quantum Process Tomography, by measuring time-averaged polarization states of the probe photon pairs.

    Depolarizing channels shrink the single-qubit Bloch sphere mapping it onto an ellipsoid. A variety of ellipsoids has been created and measured, including ellipsoids completely shrunk in one and two dimensions. The latter case corresponds, up to a unitary transformation, to bit/phase flip quantum channels.

16. Lorenzo Maccone
Quantum tomography: complete characterization of states and processes.
We give an overview of quantum tomography such that no previous knowledge on the subject is required. Starting from many copies of a system, one can reconstruct its state and the expectation value of arbitrary operators (even not observables). Techniques exist to remove some of the instrumental noise and to reduce the statistical fluctuations in the results. Maximum likelihood methods can also be employed. In addition, tomography can also reconstruct the transfer matrix of any device acting on a system (process tomography).

17. Masoud Mohseni
Direct Characterization of Quantum Dynamics
The characterization of quantum dynamics is a fundamental task in quantum mechanics, that can be accomplished via quantum process tomography (QPT). We present a general theory for direct and complete characterization of quantum dynamics of an open quantum system. In contrast to all known QPT schemes, our method relies on error-detection techniques and does not require any quantum state tomography. By analysis of the number of required experimental configurations and necessary single- and two-body quantum operations in a given Hilbert space, we demonstrate that our approach is more efficient than all known QPT schemes. We also provide a discussion of its possible physical realizations and illustrate its advantage in partial characterization of quantum dynamics.

18. Alex Monras
Optimal Phase measurements with pure Gaussian states. A statistical approach
I analyze the Heisenberg limit on phase measurements from an estimation theoretical viewpoint. I derive the attainable bounds for Gaussian states and show

that the most sensitive Gaussian state is the squeezed vacuum. I prove that homodyne detection attains the bound in the limit of large number of samples, by means of a one-step-adaptive scheme. Furthermore, I prove that no scheme without adaptivity can attain such bounds.

19. Stefano Olivares
Cloning of Gaussian states by linear optics
We describe an optical scheme for optimal Gaussian n → m cloning of coherent states. The scheme, which generalizes a recently demonstrated scheme for 1 → 2 cloning, involves only linear optical components and homodyne detection.

20. Matteo Paris
Photon statistics without photon counting and applications
Realistic photodetectors are characterized by a nonunit quantum efficiency, that is not all the photons lead to a pulse, and by a finite counting capability (resolution), i.e. a different number of detected photons lead to a discriminable output only up to a maximum value. In this talk we address in details a maximum-likelihood method assisted by measurements taken at different quantum efficiencies to reconstruct the photon statistics without a proper photon counter, and in particular using on/off avalanche photodetectors. Convergence properties as well as accuracy and robustness are analyzed numerically and experimental reconstructions of both continuous-wave and pulsed light beams are reported for semiclassical and quantum states of light, single-mode and multimode. Extension to the reconstruction of bipartite photon statistics is illustrated and experimental results are reported. Comparison among detectors with different resolution is performe showing that on/off detectors are generally enough to obtain good reconstruction and that that development of future photodetectors may be focused on increasing the quantum efficiency rather than the counting capability. Application to quantum state reconstruction are discussed and preliminary experimental results are presented.

21. Denes Petz
State estimation for qubits
Mostly the estimation of the density matrix of one or more qubits is studied. In the case of k qubits, the considered parameterization of the density matrix is given by the real and imaginary part of the entries and the parameters are estimated by independent measurements. It is established that the properties of the estimation procedure depend very much on the invertibility of the true state. In particular, for a pure state the estimation is less efficient. Moreover, several estimation schemas are compared for the unknown state of a qubit. Some of them are based of the joint measurement of several qubits. The results are illustrated by computer simulations. A question is raised and answered about the complementary reductions for two qubits.

Related papers (of the speaker):

1. D. Petz, K.M. Hangos, A. Sz´ant´o and F. Szöllősi, State tomography for two qubits using reduced densities, J. Phys. A: Math. Gen. **39**, 10901–10907, 2006.

2. D. Petz and J. Kahn, Complementary reductions for two qubits, quant-ph/0608227

3. D. Petz, K.M. Hangos and A. Magyar, Point estimation of states of finite quantum systems, in preparation

22. Massimiliano Sacchi
Information-disturbance tradeoff in estimating maximally entangled states and in quantum state discrimination
We derive the amount of information obtained by a quantum measurement that aims at estimating an unknown maximally entangled state, along with the pertaining disturbance on the state itself. A measurement instrument described by noisy maximally entangled states (and a noisy Bell measurement) is shown to provide the optimal tradeoff between such information and disturbance. The tradeoff is then studied also in the case of state discrimination between two pure quantum states. We derive the optimal tradeoff and provide the corresponding quantum measurement, which smoothly interpolates between the two limiting cases of maximal information extraction and no measurement at all.

23. Ferdinand Schmidt-Kaler
Entangled states and State tomography in an elementary ion-based quantum processor
A scalable quantum processor is discussed which based on trapped single ions. The application of gate operations leads to a generation of entangled states which are then analyzed by quantum state tomography. Multi-particle entanglement and long lived entanglement is observed. Future developements and applications of an ion based quantum processor are discussed.

24. Christof Wunderlich
Quantum state and process estimation with trapped ions